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SPACE CENTER

ANALYSIS OF TWIST AND LEAN OF TALL TOWERS

MEASUREMENT SYSTEMS DIVISION

JOHN F. KENNEDY SPACE CENTER, NASA

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TWIST AND LEAN OF TALL TOWERS

by
L. F. KEENE

ABSTRACT

This report describes a method for analytically determining the amount of twist and lean of a tall tower of equilateral triangular cross section. This method is also applicable to tall structures of other shapes and cross-sectional areas.

A BASIC computer program that determines the angle of twist and amount of lean is provided in Appendix A.

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SECTION I
INTRODUCTION

1.1 PURPOSE

This report presents an analysis of recorded data for the translational displacement of the center of mass and the rotational motion about the center of mass for the horizontal section of a tall tower.

1.2 SCOPE

This report contains illustrations showing a tall tower, diagrams of displacement measurement geometry and mechanics, and an analysis of translational and rotational displacement data.

1.3 BACKGROUND

A 150-meter tower at the Kennedy Space Center (KSC) is presently used as a structure for gathering wind data (Figure 1-1). In the course of this operation, the question arose as to the amount of tower twist and lean. Preliminary measurements established that there was a slight twist in the tower. Simultaneous measurements on the three vertices of the tower at given cross-sectional levels (Figure 1-2) were taken to determine the displacement of these vertices. By the method described in this report, these measurements were reduced to show the amount of twist and lean of the tower.

1.4 ACKNOWLEDGEMENTS

Acknowledgement for their contribution to this report is made to the personnel of LV-GDC-1 Gyro and Stabilizer Systems Branch for the simultaneous theodolite measurements.

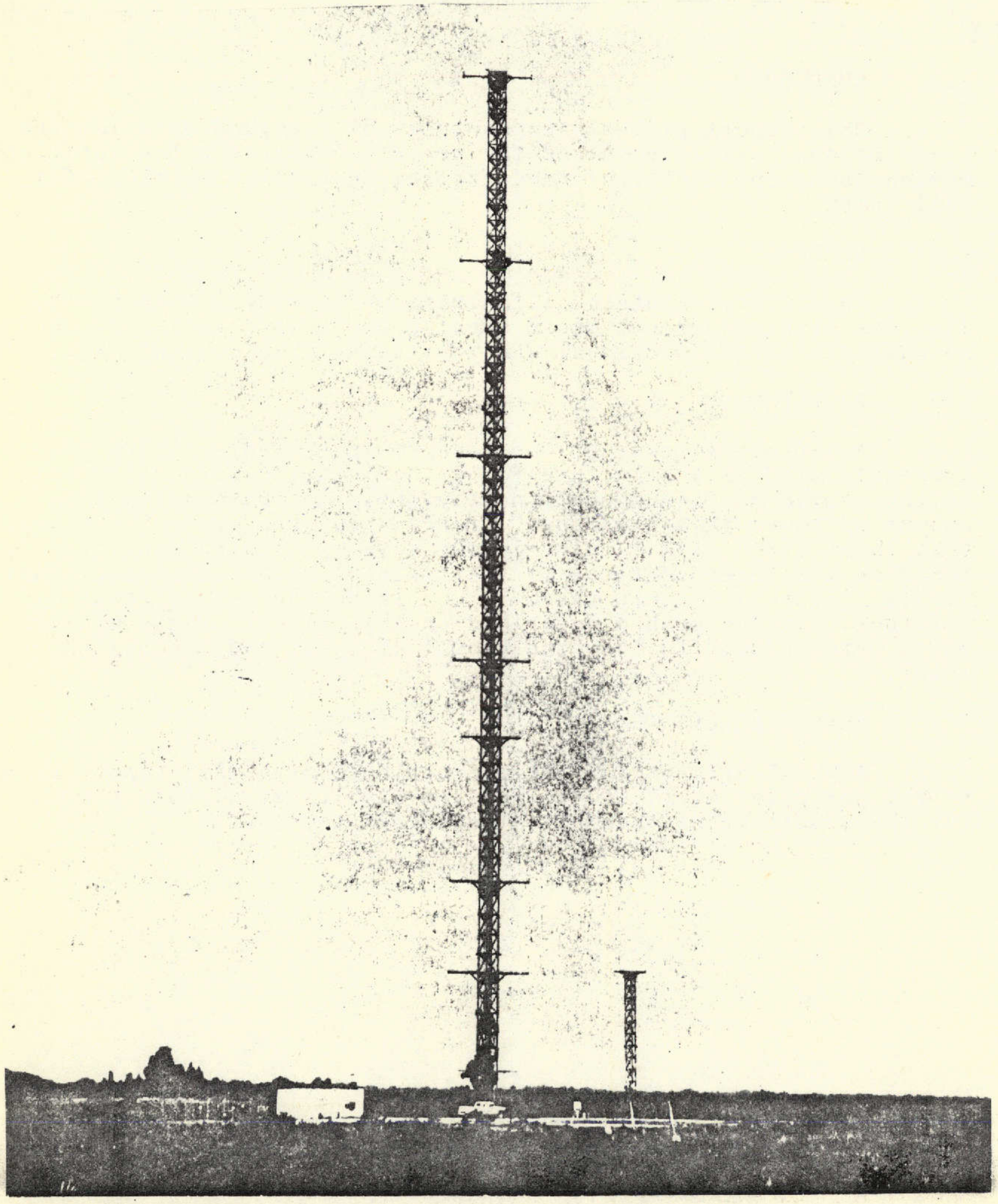

Figure 1-1. Overall View of 150-Meter Ground Winds Tower

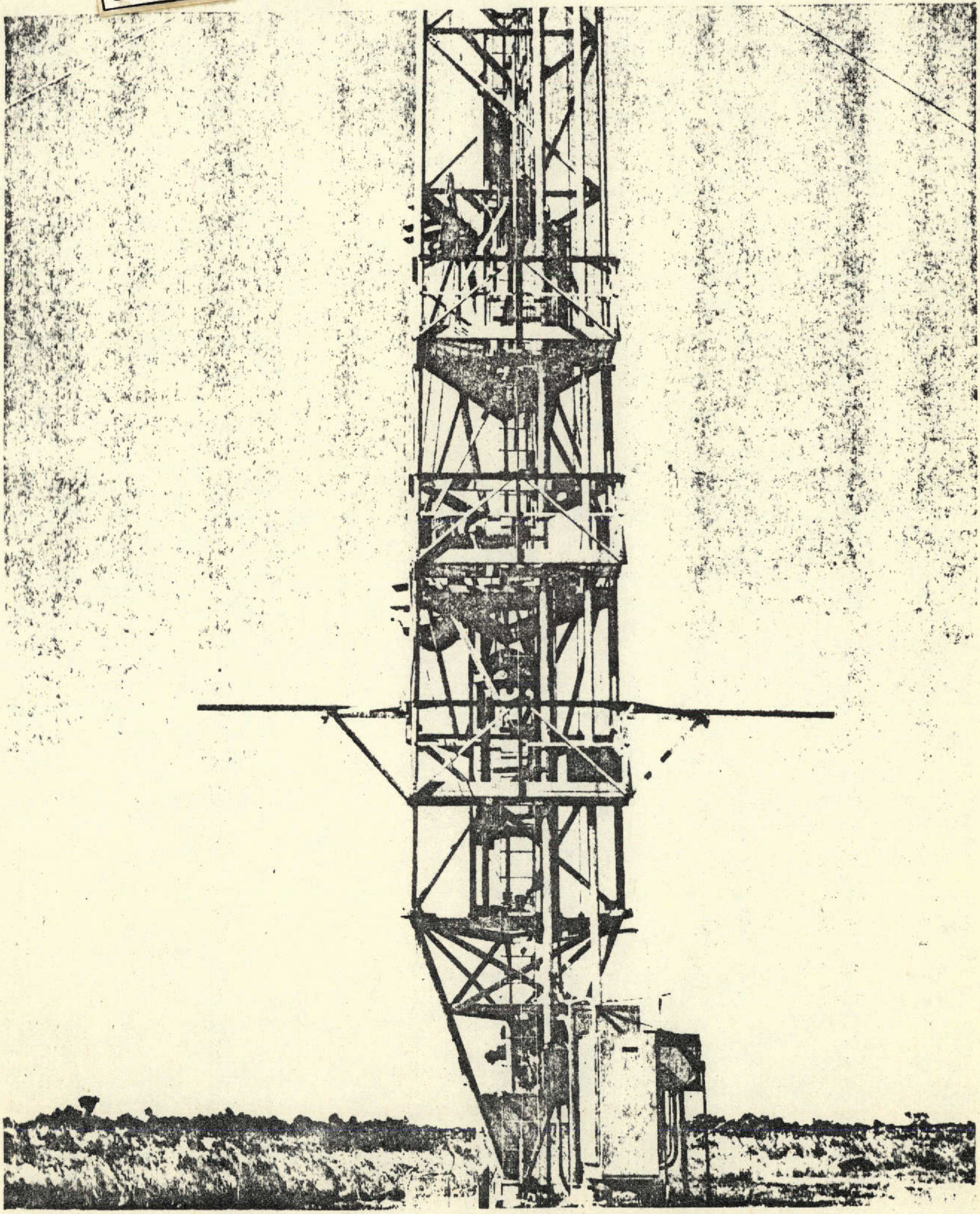

Figure 1-2. Base Section of 150-Meter Tower

SECTION II
ANALYSIS

2.1 GENERAL

The displacement of any rigid body can be described as the translational displacement of the center of mass and the rotational motion about the center of mass.

In this analysis, each horizontal section of the tower at the respective measurement level was treated as a symmetrical rigid body free to be translated or rotated only in a horizontal plane. Figure 2-1 is a schematic representation of such a horizontal section, and Figures 2-2 and 2-3 show the measuring geometry and mechanics.

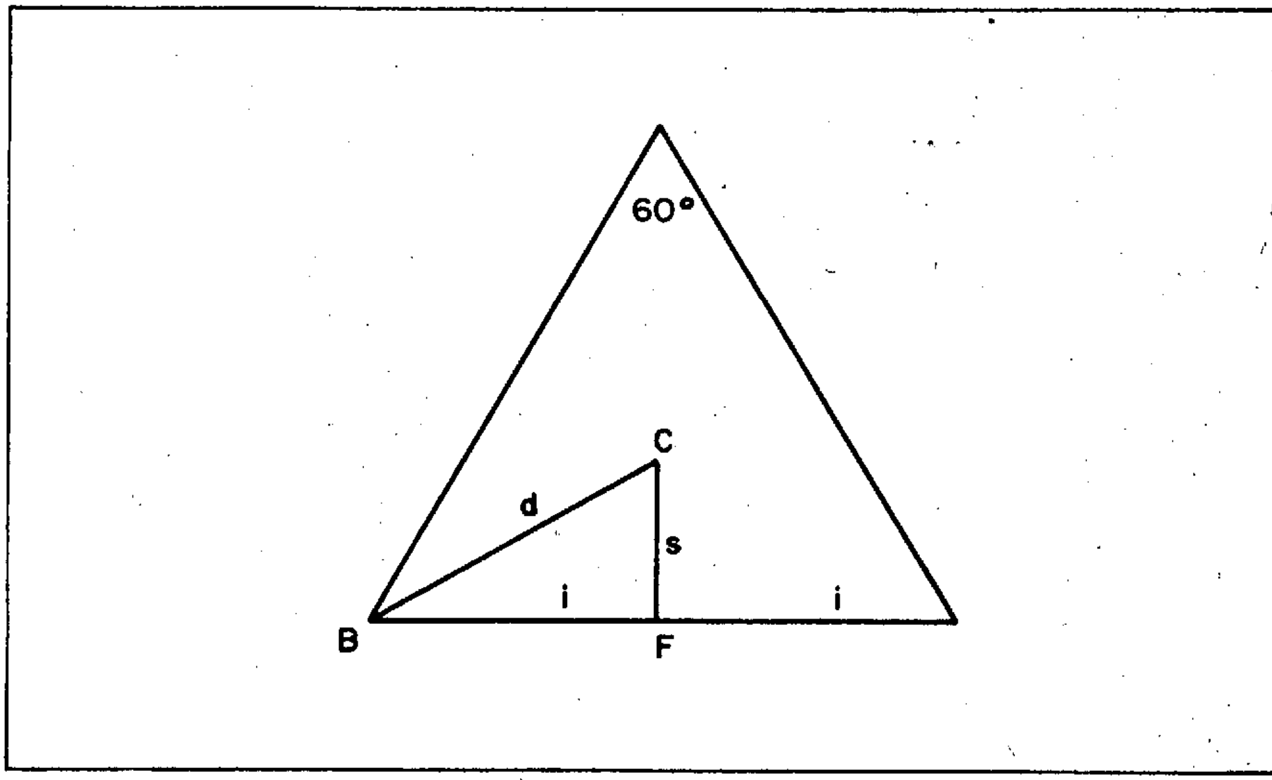

Figure 2-1. Schematic Representation of a Horizontal Section of the Tower

2.2 MEASUREMENT GEOMETRY AND MECHANICS

A theodolite was positioned at a range, r, from the tower at the intersection of the arrows (point E, Figures 2-2 and 2-3). Observations were made at the various levels of the tower, and

measurements were taken from the inside edge of the vertex strut at each level (point B, Figures 2-2 and 2-3) using the 67-foot (20.4-meter) level as the point of zero deflection. The theodolite was elevated and the angular displacement of the vertex strut at successive levels was recorded.

First, a relationship had to be established between the measured displacement at the edge of the tower and the associated relative displacement at the center of the side toward the theodolite (point E, Figures 2-2 and 2-3), due to a twist in the tower. The difference in measured displacements due to any translational motion of the tower is negligible as long as r is very much greater than i. For proof, consider Figure 2-2, where the following notation applies:

C	center of tower
2i	length of side of tower (=2BF)
Δi	measured displacement at level in question (=BD)
r	ground distance of theodolite from tower (=EF)
h	relative displacement at center of tower side (=FG)

If C moves to C' in a pure translation, then point F will move to G and point B will move to D. Since triangle BCF equals triangle DC'G, CC'=FG, and $h=\Delta i$.

now, $\tan \angle FEG = \frac{h}{r}$

$$\angle FEG = \arctan \frac{h}{r}$$

Also, $\tan \angle FEB = \frac{i}{r}$

$$\tan \angle FED = \frac{i + \Delta i}{r} = \frac{i + h}{r}$$

Therefore, $\angle FED - \angle FEB = \angle BED = \arctan \frac{i+h}{r} - \arctan \frac{i}{r}$

Now angle BED is the observed angle, while the desired angle is FEG. The difference between these angles is:

$$\Delta = \arctan \frac{i+h}{r} - \arctan \frac{i}{r} - \arctan \frac{h}{r}$$

Expanding the arc tangent functions, we obtain:

$$\Delta = \frac{i+h}{r} - \frac{1}{3}\frac{(i+h)^3}{r^3} + \frac{1}{5}\frac{(i+h)^5}{r^5} - \ldots$$

$$- \frac{i}{r} + \frac{1}{3}\frac{i^3}{r^3} - \frac{1}{5}\frac{i^5}{r^5} + \ldots$$

$$- \frac{h}{r} + \frac{1}{3}\frac{h^3}{r^3} - \frac{1}{5}\frac{h^5}{r^5} + \ldots$$

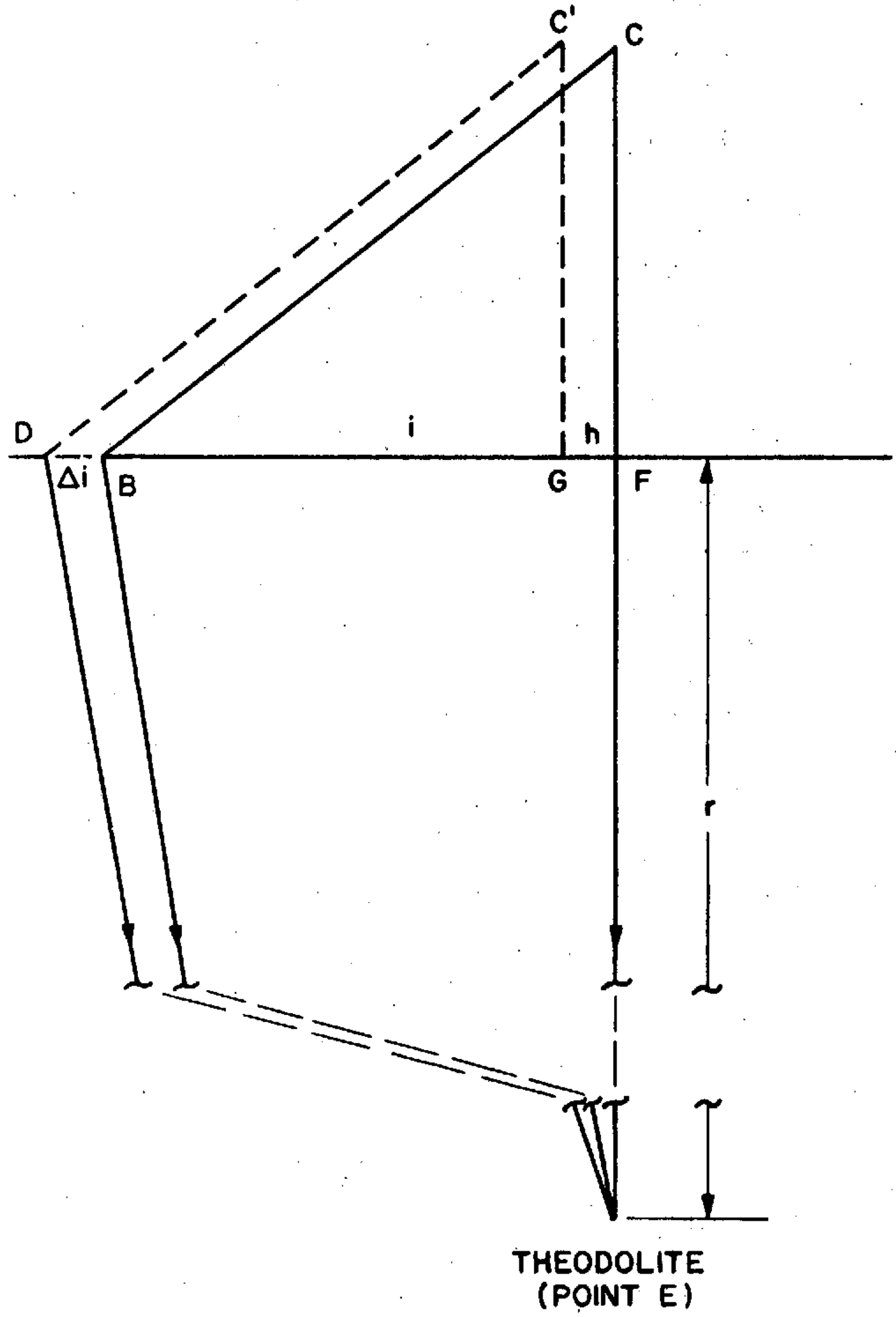

Figure 2-2. Translational Displacement Measurement Geometry and Mechanics

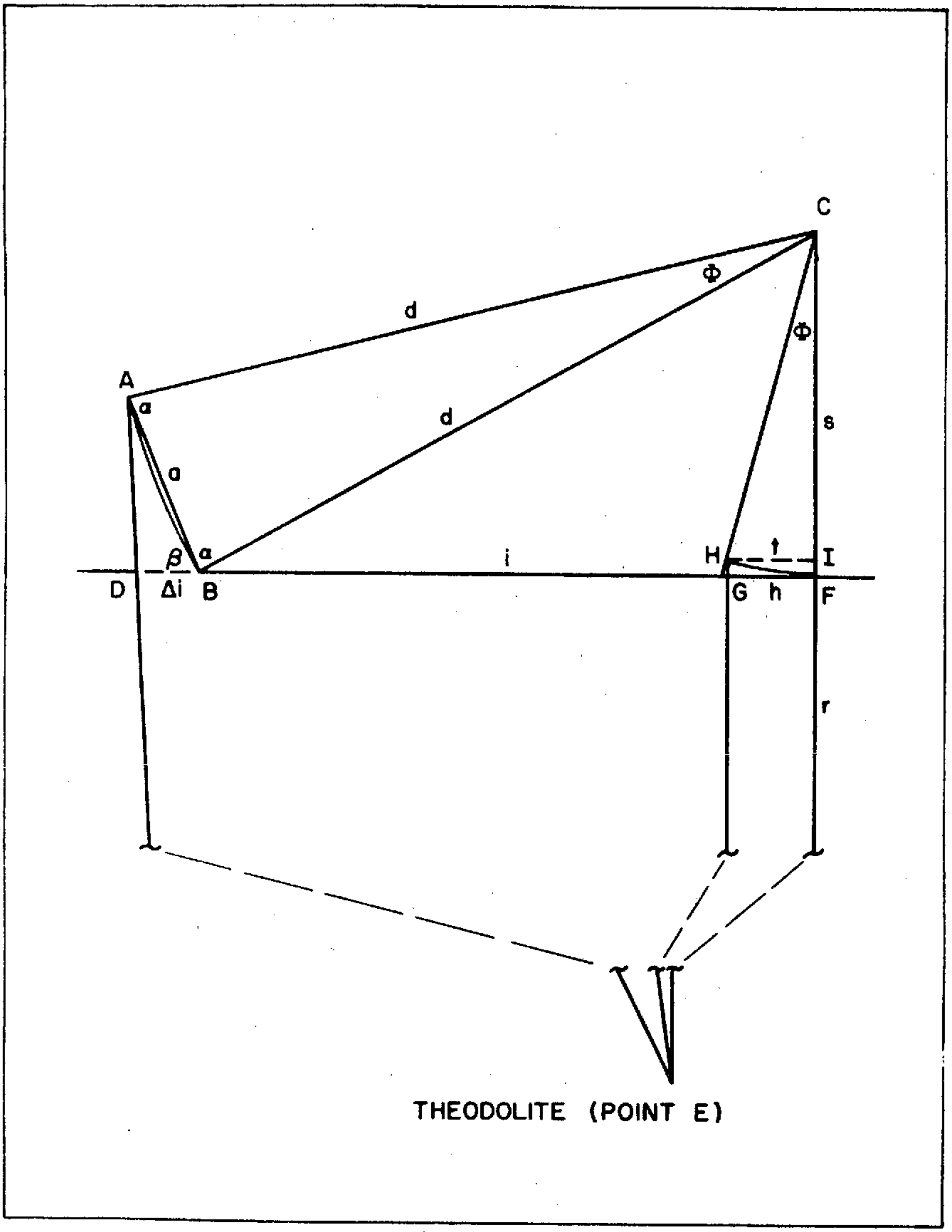

Figure 2-3. Rotational Displacement Measurement Geometry and Mechanics

All terms involving h and i alone vanish, leaving:

$$\Delta=-\frac{1}{3}\frac{3i^2h+3ih^2}{r^3}+\frac{1}{5}\frac{5i^4h+10i^3h^2+10i^2h^3+5ih^4}{r^5}$$

$$-\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot$$

The resulting error in the measurement of h would be:

$$\Delta\cdot r=-\frac{i^2h+ih^2}{r^2}+\frac{i^4h+2i^3h^2+2i^2h^3+ih^4}{r^4}-\cdot\ \cdot\ \cdot\ \cdot\ \cdot$$

Let $h = 1$, $i = 4$, $r = 400$, then:

$$\Delta\cdot r=-\frac{20}{16x10^4}+\frac{422}{256x10^8}=-1.25x10^{-4}+1.65x10^{-8}$$

$$\Delta\cdot r\cong-1.25x10^{-4}\cong-\frac{1}{8000}$$

This is the resultant error when h is subsituted for Δi in the example.

In Figure 2-3, we have the following notation:

C	center of tower
2i	length of side of tower
Δi	measured displacement at level in question (=BD)
h	relative displacement at center of tower side (=FG)
t	derived displacement at center of tower side (=IH) (used later in analysis)
d	distance from center to vertex (=BC)
s	distance from center to side (=CF)
Φ	twist angle
r	ground distance of theodolite from tower (=EF)

The mechanics of Figure 2-3 are explained as follows. As the tower twists, the line CF moves through an angle Φ to position CH. The diagonal CB moves through the same angle Φ to position CA. Since the theodolite measures angles, the instrument sees a displacement from point B to point D, which is the point where the line AE intersects the extended line of the tower side. If the theodolite had a reference point F at the center of the tower, it would see a displacement from point F to point G which is the intersection of HE with the tower side. First, a relationship must be found between BD, i.e. Δi, and t, i.e. s sin Φ (=IH), which latter quantity is used in the analysis, and finally a relationship between t and h (=FG).

From Figure 2-1 we have the following relationships:

$$\cos 30° = \frac{i}{d}, \quad \tan 30° = \frac{s}{i}$$

$$d = \frac{i}{\cos 30°} = \frac{2}{\sqrt{3}} \qquad s = i \tan 30° = \frac{i}{\sqrt{3}} \qquad d = 2s$$

From Figure 2-3,

$$t = s \sin \Phi$$

From the law of cosines, triangle ABC,

$$a^2 = 2d^2 - 2d^2 \cos \Phi = 2d^2(1-\cos \Phi)$$

$$a = \sqrt{2}d \cdot \sqrt{1-\cos \Phi} = \sqrt{2}d\sqrt{\frac{2(1-\cos \Phi)}{2}}$$

$$a = 2d \sin \frac{\Phi}{2} = 4s \sin \frac{\Phi}{2}$$

Also,

$$2\,\angle CBA + \Phi = 180°$$

$$\angle CBA = 90° - \frac{\Phi}{2} = \alpha$$

In triangle ABD,

$$\angle ABD + \angle CBA + 30° = 180°$$

$$\angle ABD = 180 - 30° - \angle CBA = 180° - 30° - 90° + \frac{\Phi}{2}$$

$$\angle ABD = 60° + \frac{\Phi}{2} = \beta$$

In triangle ACE, from the law of tangents,

$$\tan \frac{(\angle CAD - \angle AEC)}{2} = \frac{r+s-d}{r+s+d} \cot \frac{(60° + \Phi)}{2}$$

$$\angle CAD - \angle AEC = 2 \text{ arc tan}\left[\frac{r+s-d}{r+s+d} \cot \frac{(60° + \Phi)}{2}\right] = 2 \text{ arc tan}\left[\frac{r-s}{r+3s} \cot \frac{60° + \Phi}{2}\right]$$

$$\angle CAD - \angle AEC = 2K, \text{ where arc tan}\left[\frac{r-s}{r+3s} \cot \frac{60° + \Phi}{2}\right] = K$$

Again in triangle ACE,

$$\angle CAD + \angle AEC + 60° + \Phi = 180°$$

$$\angle AEC = 120° - \Phi - \angle CAD$$

$$\angle CAD - 120° + \Phi + \angle CAD = 2K$$

$$2\angle CAD = 2K + 120° - \Phi$$

$$\angle CAD = K + 60° - \frac{\Phi}{2}$$

Also,

$$\angle DAB = \angle CAD - \alpha = K + 60° - \frac{\Phi}{2} - 90° + \frac{\Phi}{2}$$

$$\angle DAB = K - 30°$$

$$\angle ADB + \beta + \angle DAB = 180°$$

$$\angle ADB = 180 - \beta - K + 30° = 180° - 60° - \frac{\Phi}{2} - K + 30°$$

$$\angle ADB = 150° - \frac{\Phi}{2} - K$$

In triangle ADB, by the law of sines,

$$\frac{a}{\sin\angle ADB} = \frac{\Delta i}{\sin\angle DAB}$$

$$\Delta i = \frac{a\sin(K-30°)}{\sin\left(150° - \frac{\Phi}{2} = K\right)}$$

$$\Delta i = 4s\sin\frac{\Phi}{2}\left[\frac{\sin(K-30°)}{\sin\left(150° - \frac{\Phi}{2} - K\right)}\right]$$

Now $t = s\sin\Phi = s\sin 2\frac{\Phi}{2}$

$$t = 2s\sin\frac{\Phi}{2}\cos\frac{\Phi}{2}$$

$$2t = 4s \sin \frac{\Phi}{2} \cos \frac{\Phi}{2}$$

$$4\ s \sin \frac{\Phi}{2} = \frac{2t}{\cos \frac{\Phi}{2}}$$

Substituting in the expression for Δi,

$$\Delta i = \frac{2t}{\cos \frac{\Phi}{2}} \left[\frac{\sin\ (K-30°)}{\sin\ (150° - \frac{\Phi}{2} - K)}\right]$$

$$\frac{t}{\Delta i} = \frac{1}{2} \cos \frac{\Phi}{2} \left[\frac{\sin\ (150° - \frac{\Phi}{2} - K)}{\sin\ (K - 30°)}\right]$$

Figure 2-4 is a plot of $\frac{t}{\Delta i}$ versus Φ, using the following known quantities:

r = 400 feet
i = 3.833 feet
s = 2.213 feet
d = 4.426 feet

Use will be made of this graph (Figure 2-4) later. (It will be noted that for $\Phi = 0$, the problem is not defined.)

In order to determine how much t differs from h, we proceed as follows:

In triangle FEG, $\tan \angle FEG = \frac{h}{r}$

also, $\tan \angle FEG = \tan \angle IEH = \frac{t}{r + (s - s \cos \Phi)}$

$$\frac{h}{r} = \frac{t}{r + (s - s \cos \Phi)}$$

$$\frac{h}{t} = \frac{r}{r + s\ (1 - \cos \Phi)}$$

Now, $\sin^2 \frac{\Phi}{2} = \frac{1 - \cos \Phi}{2}$

therefore, $\frac{h}{t} = \frac{r}{r + 2s \sin^2 \frac{\Phi}{2}}$

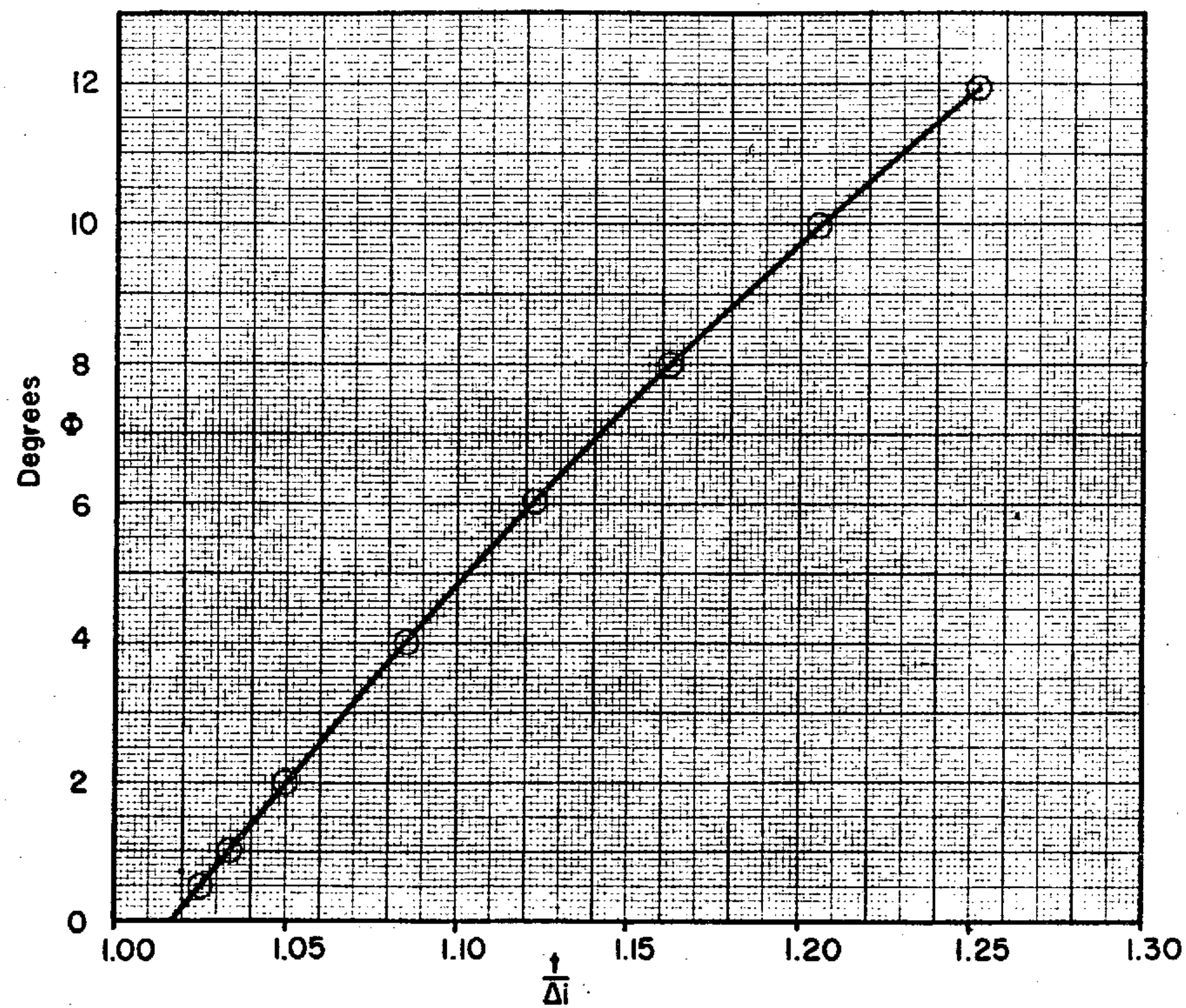

Figure 2-4. Plot of $\frac{t}{\Delta i}$ Versus Φ

If $r = 400$ feet
$s = 2.213$ feet
$\Phi = 10$ degrees

We have

$$\frac{h}{t} = \frac{400}{400 + 4.426\ (.0075969)} = \frac{400}{400.03362}$$

$$\frac{h}{t} = 0.99992$$

Therefore in this case, h differs from t by 1 part in 12,000.

2.3 MEASUREMENT ANALYSIS

We can now determine the twist angle and the displacement of a given level of the tower from the three thodolite measurements. Let us first define a quantity $\Delta i'$, the actual measured displacement, which is the sum of the displacements due to translation and rotation. Also we define a quantity t' which is the sum of the t's due to translation and rotation. In other words, each $\Delta i'$ can be considered as being the sum of three components: a component due to the x displacement, a component due to the y displacement, and a component due to twist. In like manner, each t' can be considered as being made up of three components: a component due to the x displacement, a component due to the y displacement, and a component due to twist. The components due to the horizontal displacements are the same, (with negligible error) for both $\Delta i'$ and t', while the components due to twist are related by the function graphed in Figure 2-4.

Now consider Figure 2-5 where the total derived displacement, that is, the t's for each theodolite are shown. The x displacements are positive to the right, y displacements are positive upward, and angles are positive clockwise. We may write the following two sets of equations:

$$t_1' = 0\cdot\Delta x + \Delta y + s \sin \Phi$$
$$t_2' = -\Delta x \sin 60° - \Delta y \sin 30° + s \sin \Phi$$
$$t_3' = \Delta x \sin 60° - \Delta y \sin 30° + s \sin \Phi$$

$$\Delta i_1' = 0\cdot\Delta x + \Delta y + \Phi\ (\Phi)\cdot s \sin \Phi$$
$$\Delta i_2' = -\Delta x \sin 60° - \Delta y \sin 30° + \Phi\ (\Phi)\cdot s \sin \Phi$$
$$\Delta i_3' = \Delta x \sin 60° - \Delta y \sin 30° + \Phi\ (\Phi)\cdot s \sin \Phi$$

where $\Phi\ (\Phi)$ stands for some function of Φ.

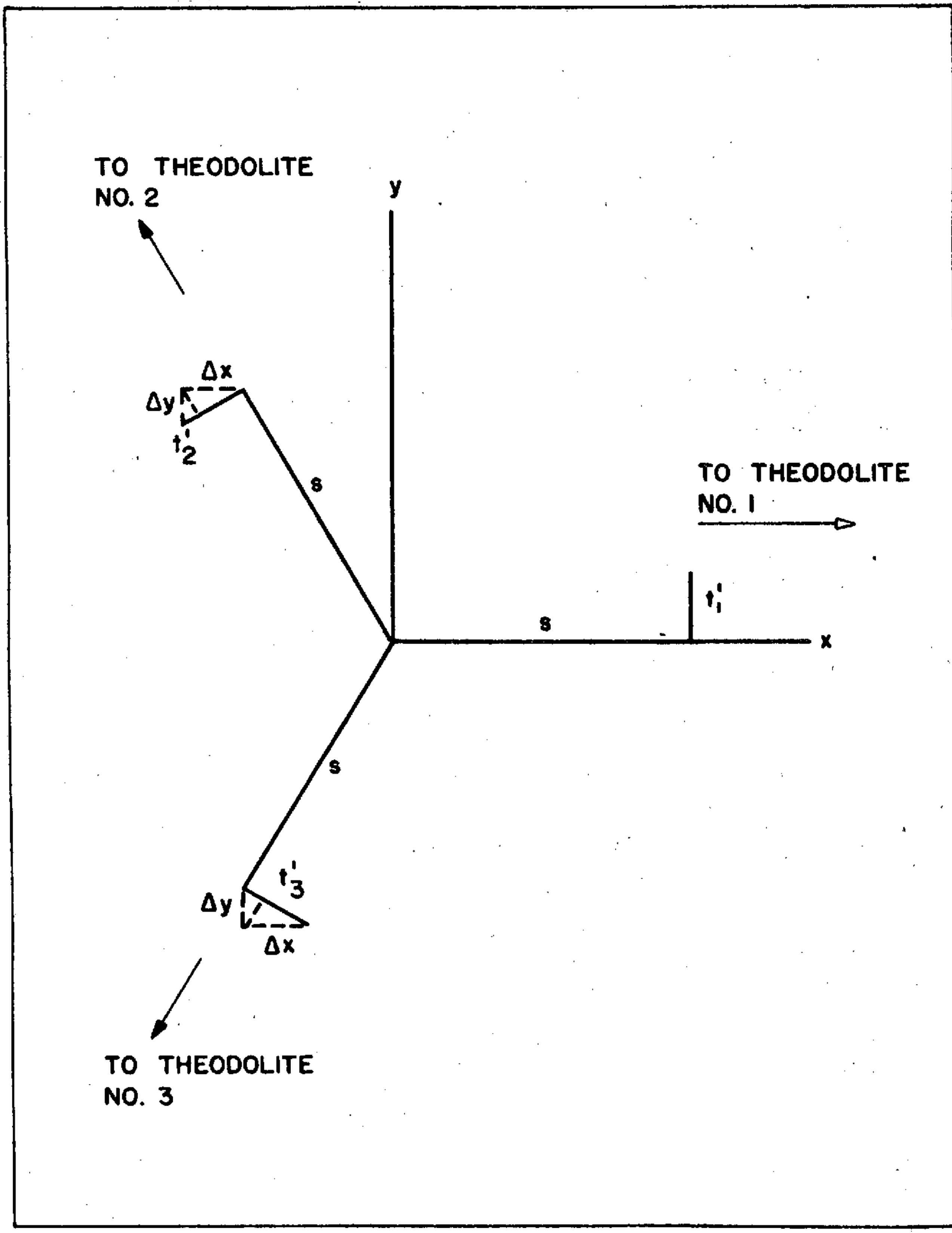

Figure 2-5. Tower Displacement Components

The second set of equations constitute three simultaneous equations in three unknowns, and the solution is readily obtainable by determinants. The unknowns are Δx, Δy, and sin Φ, and the known terms are $\Delta i_1'$, $\Delta i_2'$, and $\Delta i_3'$. The determinant D of the coefficients of the unknowns is:

$$D = \begin{vmatrix} 0 & 1 & S \\ -\sqrt{3}/2 & -0.5 & S \\ +\sqrt{3}/2 & -0.5 & S \end{vmatrix} = S \begin{vmatrix} 0 & 1 & 1 \\ -\sqrt{3}/2 & -0.5 & 1 \\ +\sqrt{3}/2 & -0.5 & 1 \end{vmatrix}$$

Therefore, Δx can be written:

$$\Delta x = \frac{1}{D} \begin{vmatrix} \Delta i_1' & 1 & S \\ \Delta i_2' & -0.5 & S \\ \Delta i_3' & -0.5 & S \end{vmatrix} = \frac{S}{D} \begin{vmatrix} \Delta i_1' & 1 & 1 \\ \Delta i_2' & -0.5 & 1 \\ \Delta i_3' & -0.5 & 1 \end{vmatrix}$$

where $S = \Phi(\phi) \cdot s$

$$\Delta x = \frac{2}{3\sqrt{3}} (-0.5\ \Delta i_2' + 0.5\ \Delta i_3' + 0.5\ \Delta i_1' + \Delta i_3' - 0.5\ \Delta i_1' - \Delta i_2')$$

$$\Delta x = \frac{-3\ \Delta i_2' + 3\ \Delta i_3'}{3\sqrt{3}} = \frac{\Delta i_3' - \Delta i_2'}{\sqrt{3}}$$

$$\Delta x = \frac{\sqrt{3}}{3} (\Delta i_3' - \Delta i_2')$$

Likewise for Δy:

$$\Delta y = \frac{1}{D} \begin{vmatrix} 0 & \Delta i_1' & S \\ -\sqrt{3}/2 & \Delta i_2' & S \\ +\sqrt{3}/2 & \Delta i_3' & S \end{vmatrix} = \frac{S}{D} \begin{vmatrix} 0 & \Delta i_1' & 1 \\ -\sqrt{3}/2 & \Delta i_2' & 1 \\ +\sqrt{3}/2 & \Delta i_3' & 1 \end{vmatrix}$$

$$\Delta y = \frac{2}{3\sqrt{3}} (-\sqrt{3}/2\ \Delta i_3' - \sqrt{3}/2\ \Delta i_2' + \sqrt{3}/2\ \Delta i_1' + \sqrt{3}/2\ \Delta i_1')$$

$$\Delta y = \frac{2\ \Delta i_1' - \Delta i_2' - \Delta i_3'}{3}$$

And for $\Phi\ (\Phi) \sin \Phi$,

$$\Phi\ (\Phi) \sin \Phi = \frac{1}{D} \begin{vmatrix} 0 & 1 & \Delta i_1' \\ -\sqrt{3}/2 & -0.5 & \Delta i_2' \\ +\sqrt{3}/2 & -0.5 & \Delta i_3' \end{vmatrix}$$

$$\Phi\ (\Phi) \sin \Phi = \frac{2}{3s\sqrt{3}} (\sqrt{3}/2\ \Delta i_3' + \sqrt{3}/2\ \Delta i_2' = \sqrt{3}/4\ \Delta i_1' + \sqrt{3}/4\ \Delta i_1')$$

$$\Phi\ (\Phi) \sin \Phi = \frac{\Delta i_1' + \Delta i_2' + \Delta i_3'}{3s}$$

$$\sin \Phi = \frac{f\ (\Phi)}{3s} (\Delta i_1' + \Delta i_2' + \Delta i_3')$$

$$\text{where } f\ (\Phi) = \frac{1}{\Phi\ (\Phi)}$$

$$\Phi = \text{arc} \sin \left[\frac{f\ (\Phi)}{3s} (\Delta i_1' + \Delta i_2' + \Delta i_3') \right]$$

For the NASA 150-Meter Ground Winds Tower, s = 2.213 feet, and therefore:

$$\Delta x = 0.577\ (\Delta i_3' - \Delta i_2')$$

$$\Delta y = 0.333\ (2\ \Delta i_1' - \Delta i_2' - \Delta i_3')$$

$$\Phi = \text{arc} \sin\ [0.01255\ f\ (\Phi)\ (\Delta i_1' + \Delta i_2' + \Delta i_3')]$$

where the coefficient 0.01255 is in inches^{-1}.

It will be noted that the correct value of Φ, the twist angle, must be known before f (Φ) can be found. However f (Φ) can be readily found from Figure 2-4 by successive approximations, assuming an initial, arbitrary, value of Φ. An example will make the method clear. Some actual tower measurements are shown in Table 2-1.

Table 2-1. Tower Twist and Lean Measurements

Level	Altitude (ft)	$\Delta i_1'$ (in.)	$\Delta i_2'$ (in.)	$\Delta i_3'$ (in.)
1 (Ref)	67.10	0	0	0
2	133.70	-1.08	-0.24	-0.94
3	192.10	-1.49	-0.40	-2.12
4	250.40	-1.51	-0.19	-1.93
5	308.70	-1.95	+0.21	-4.10
6	367.10	-2.66	+0.09	-5.30
7	425.40	-3.30	+0.66	-6.22
8	483.73	-3.71	+0.85	-7.11

Let us assume a value for Φ of 5 degrees at level 8. From the graph of Figure 2-4, we obtain a value for $f(\Phi)$ of 1.104.

This will give a new value for Φ of,

$$\Phi = \text{arc sin } (0.01255 \times 1.104)\ (-3.71 + 0.85 - 7.11)$$

$$\Phi = \text{arc sin } (0.013855)\ (-9.97)$$

$$\Phi = \text{arc sin } (-0.13813)$$

$$\Phi = -7°56'23'' = -7.94°$$

Again entering the graph of Figure 2-4 we obtain a conversion factor of 1.162, therefore,

$$\Phi = \text{arc sin } (0.01255 \times 1.162)\ (-9.97)$$

$$\Phi = \text{arc sin } (0.014583)\ (-9.97)$$

$$\Phi = \text{arc sin } (-0.14539)$$

$$\Phi = -8°21'36'' = -8.36°$$

Repeating the preceding process,

$$\Phi = \text{arc sin } (0.01255 \times 1.171)\ (-9.97)$$

$$\Phi = \text{arc sin } (0.014696)\ (-9.97)$$

$$\Phi = \text{arc sin } (-0.14652)$$

$$\Phi = -8°25'31'' = -8.42°$$

A final iteration gives the required value,

$$\Phi = \text{arc sin } (0.01255 \times 1.172)\ (-9.97)$$

$$\Phi = \text{arc sin } (0.014709)\ (-9.97)$$

$$\Phi = \text{arc sin } (-0.14665)$$

$$\Phi = -8°25'58'' = -8.43°$$

Table 2-2 shows the calculated values of the displacements and twists.

Table 2-2. Tower Displacement and Twist Angle Calculated Values

Level	Δx (in.)	Δy (in.)	Φ (degrees)
1	0	0	0
2	-0.40	-0.33	-1.69
3	-0.99	-0.15	-3.08
4	-1.00	-0.30	-3.02
5	-2.49	0	-4.61
6	-3.11	-0.04	-6.41
7	-3.97	-0.35	-7.34
8	-4.59	-0.39	-8.43

As a check on the calculations, a graphical overlay model was constructed of the top level and the ground level. The twist angle as measured with a protractor was 8.5 degrees, and the displacement of the tower center was measured as 4.68 inches as against the calculated 4.61 inches. These results are in excellent agreement.

The analysis of the last section is directly applicable to structures of circular cross section. If three fiducial marks are equispaced around the structure (point F, Figure 2-6, for one mark), then the displacement and twist of any cross section may be determined from the preceding equations for Δx, Δy, and Φ. It is assumed, in this case, that the theodolite is pointed normally to the tangent at the fiducial mark.

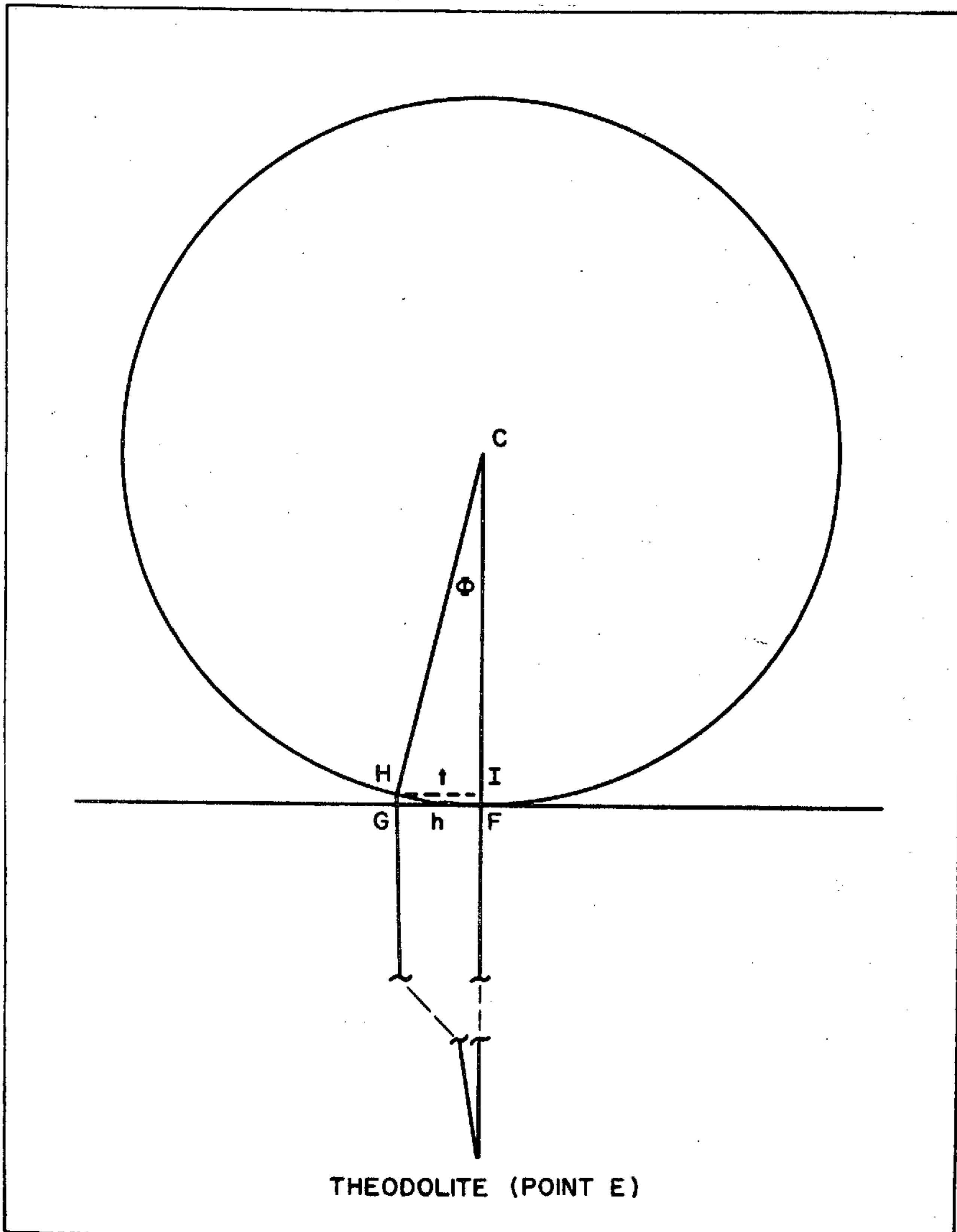

Figure 2-6. Structure of Circular Cross Section

APPENDIX A

COMPUTER PROGRAM

A program has been written in BASIC for use with the GE-635 computer time sharing system. The following notations have been used in the program:

$\Delta i_1' = I_1$

$\Delta i_2' = I_2$

$\Delta i_3' = I_3$

Φ = P (initial value)

K = FNA (P)

$f(\Phi)$ = FNC (P)

Δx = X

Δy = Y

Φ = P3 (final value)

LEVEL = L

The values used in the data steps are the values used in the sample calculations. Any values of course could be used.

```
010 REM THIS PROGRAM DETERMINES THE ANGLE OF TWIST AND AMOUNT OF LEAN
020 REM IN HORIZONTAL X & Y OF A TRIANGULAR TOWER WHERE S IS THE
030 REM DISTANCE FROM THE CENTER TO ONE SIDE.  THREE THEODOLITES
040 REM AT THE SAME DISTANCE R FROM THE CENTER OF EACH SIDE MEASURE THE
050 REM HORIZONTAL DISPLACEMENTS I1, I2, & I3 OF THE CORNERS OF THE
060 REM TOWER RELATIVE TO SOME REFERENCE LEVEL.  (SEE KSC TR-297, REV 1)
070 PRINT SPC(0) "LEVEL" SPC(9)"X" DISPLACEMENT" SPC(9)"Y" DISPLACEMENT";
080 PRINT SPC(9) "TWIST ANGLE"
090 PRINT SPC(2)"#"SPC(18)"IN."SPC(20)"IN."SPC(17)"DEG."
100 READ R,S
110 READ L, I1, I2, I3
120 DEF FNA(P) = ATN((R-S)*COT((60*.0174533+P)/2)/(R+3*S))
130 DEF FNB(P) = .5*COS(P/2)*SIN(150*.0174533-P/2-FNA(P))
140 DEF FNC(P) = FNB(P)/SIN(FNA(P)-30*.0174533)
150 REM AT THIS POINT AN ESTIMATE IS MADE AS TO THE APPROXIMATE VALUE OF
160 REM P, THE TWIST ANGLE.  IN THIS CASE P IS ESTIMATED TO BE 5
170 REM DEGREES, OR 0.87266 RADIAN.
180 LET P = .087266
190 LET F = FNC(P)
200 REM P1 = ARC SIN(F*(I1+I2+I3)/(3*S*12))
210 REM NO ARC SIN FUNCTION IN BASIC.
220 REM ARC SIN(G) = ATN(G/SQR(1-G↑2))
230 LET G=F*(I1+I2+I3)/(3*S*12)
240 LET P1=ATN(G/SQR(1-G↑2))
250 LET P2=ABS(P1)
260 IF P=P2 THEN 290
270 LET P=P2
280 GO TO 190
290 LET X=(1/SQR(3))*(I3-I2)
300 LET Y=(1/3)*(2*I1-I2-I3)
310 LET P3=P1*57.2958
320 PRINT USING 330,L,X,Y,P3
330: ##                  ###.##                  ###.##                  ###.##

340 GO TO 110
350 DATA 400,2.213
360 DATA 1,0,0,0,2,-1.08,-.24,-.94,3,-1.49,-.40,-2.12,4,-1.51,-.19
370 DATA -1.93,5,-1.95,+.21,-4.10,6,-2.66,+.09,-5.30
380 DATA 7,-3.30,+.66,-6.22,8,-3.71,+.85,-7.11
390 END
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APPROVAL

ANALYSIS OF TWIST AND LEAN OF TALL TOWERS

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